The anomalous lepton magnetic moment, LFV decays and the fourth generation

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Abstract

We investigate the lepton flavor violation (LFV) decays, $\tau \to l\gamma$ $(l = \mu, e)$ and $\mu \to e\gamma$, and the newly observed muon $g-2$ anomaly in the framwork of a squential fourth generation model with a heavy fourth neutrino, ν'. By using the recent experimental bounds, we take the constraints of the 4×4 leptonic mixing matrix element factors, $|V_{1\nu'}V_{2\nu'}|^2$, $|V_{1\nu'}V_{3\nu'}|^2$ and $|V_{3\nu'}V_{2\nu'}|^2$. We find that LFV decays and $g_\mu - 2$ can exclude most of the parameter space of the 4th generation neutrino mass $m_{\nu'}$ and give stringent constraints on the existence of the fourth generation.

At present, Standard model (SM) has to face the experimental difficaties which are all relate to leptons. It seems to indicate the presence of new physics just round the cornor will be in the leptonic part. Firstly, there are convincing evdences that neutrinos are massive and oscillate in flavor [1]. Secondly, the recent measurment of the muon anomalous magnetic moment by the experiment E821 [2] at Brookhaven National Laboratory disagrees with the SM expectations at more than 2.6σ level (the new deviation drops to to 1.6σ[3]).

Defined as $a_\mu \equiv (g_\mu - 2)/2$, the recent measurement of a_μ is

$$a_\mu^{\rm exp} = (11\,659\,202 \pm 14 \pm 6) \times 10^{-10}, \tag{1}$$

where the SM prediction is

$$a_\mu^{\rm SM} = (11\,659\,159.7 \pm 6.7) \times 10^{-10}, \tag{2}$$

Thus, one finds,

$$\delta a_\mu^{\rm SM} \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = (42.6 \pm 16.5) \times 10^{-10} \tag{3}$$

which gives the old 2.6σ deviation. But the revised difference between experiment and SM is

$$\delta a_\mu^{\rm SM} \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = 26(16) \times 10^{-10} \tag{4}$$

which is now only a 1.6σ deviation. Although the deviation drops from 2.6σ to 1.6σ, it might seem that the absolute magnitude of the deviation may be a hint of new physics. There have been a lot of scenarios of new physics proposed to interpret the non-vanishing and positive value of δa_μ. Many of these new physics effects can also contribute to the lepton flavor violation processes, such as $\tau \to l\gamma$ $(l = \mu, e)$ and $\mu \to e\gamma$. In this note, we consider the sequential fourth generation standard model (SM4) to investigate its contributions to a_μ and some leptonic flavor violation decays.

From the point of phenomenology, there is a realistic question what are numbers of the fermions generation or weather there are other additional quarks or leptons. The present experiments tell us there are only three generation fermions with *light* neutrinos which mass are less smaller than $M_Z/2$[4]. But the experiments don't exclude the existence of other additional generation, such as the fourth generation, with a *heavy* neutrino, i.e. $m_{\nu_4} \geq M_Z/2$[5]. Many refs. have studied the SM4 [6], which is added an up-like quark $t^{'}$, a down-like quark $b^{'}$, a lepton $\tau^{'}$, and a heavy neutrino $\nu^{'}$ in the SM. The properties of these new fermions are all the same as their corresponding counterparts of other three generations except their masses and CKM mixing, see Tab.1,

If there exists a very heavy fourth neurino ν', it can contribute to a_μ and lepton flavor violation decays through diagram of Fig. 1. This is the electroweak interaction. Similar to that of quarks, the corresponding Lagragian is

$$\mathcal{L} = -\frac{g}{\sqrt{2}}(\bar{\nu}'\gamma_\mu a_L V_{l\nu'} l)W^\mu + h.c. \tag{5}$$

	up-like quark	down-like quark	charged lepton	neutral lepton
SM fermions	u c t	d s b	e μ τ	ν_e ν_μ ν_τ
new fermions	$t^{'}$	$b^{'}$	$\tau^{'}$	ν'

Table 1: The elementary particle spectrum of SM4

where $a_L = (1-\gamma_5)/2$, $V_{l\nu'}(l = e\mu, \tau)$ is the $(1,4), (2,4)$ and $(3,4)$ elements of the four-generaton lepton mixing MNS matrix (4×4),

$$V_{\mathrm{MNS}}^{\mathrm{SM4}} = \begin{pmatrix} V_{1\nu_e} & V_{1\nu_\mu} & V_{1\nu_\tau} & V_{1\nu'} \\ V_{2\nu_e} & V_{2\nu_\mu} & V_{2\nu_\tau} & V_{2\nu'} \\ V_{3\nu_e} & V_{3\nu_\mu} & V_{3\nu_\tau} & V_{3\nu'} \\ V_{4\nu_e} & V_{4\nu_\mu} & V_{4\nu_\tau} & V_{4\nu'} \end{pmatrix} \tag{6}$$

Reverting back to the diagrams of Fig. 1, we see that the fourth neutrino contribution to decays $\tau \to e\gamma, \mu\gamma$ and $\mu \to e\gamma$ (see Fig. 1). We don't consider the three usual neutrinos' contribution because their masses are too small mass and have almost no effects on LFV decays and anomalous magnetic moment of leptons. Because of its heavy mass, the 4th neutrino ν' in SM4 can induce these decays. Calculating Fig. 1, we obtain

$$\Gamma(\tau \to \mu\gamma) = \frac{\alpha G_F^2 m_\tau^5}{96} |V_{2\nu'} V_{3\nu'}|^2 f^2(x). \tag{7}$$

where $x \equiv m_{\nu'}^2/m_W^2$, α is the fine constrcture constant and

$$f(x) = \frac{-5x^3 - 5x^2 + 4x}{12(x-1)^3} + \frac{(2x^3 - x^2)\log x}{2(x-1)^4}. \tag{8}$$

Simlarly, we obtain

$$\Gamma(\tau \to e\gamma) = \frac{\alpha G_F^2 m_\tau^5}{96} |V_{1\nu'} V_{3\nu'}|^2 f^2(x), \tag{9}$$

$$\Gamma(\mu \to e\gamma) = \frac{\alpha G_F^2 m_\mu^5}{96} |V_{2\nu'} V_{1\nu'}|^2 f^2(x). \tag{10}$$

Using the current experimental bounds of these three LFV processes [7], we obtain the parameter space of other 4×4 matrix elements of leptonic mixing, (see Fig. 2). From Figs. 2, considering the unitarity of the matrix V_{MNS}^{SM4}, we find the reasonable range of m'_μ is under the curve.

The Lagragian related to $g_\mu - 2$ is

$$\mathcal{L} = -\frac{g}{\sqrt{2}} (\bar{\nu}' \gamma_\mu a_L V_{2\nu'} \mu) W^\mu + h.c. \tag{11}$$

From this Lagragian, we see that the fourth neutrino contribution to a_μ is

$$a_\mu^{\rm SM4} \;=\; \alpha(\frac{g}{\sqrt{2}}V_{2\nu'})^2\frac{2m_\mu^2}{m_W^2}\cdot f(x). \tag{12}$$

We suppose that the 1.6σ discrepancy of muon anomalous magnetic moment, $\delta a_\mu^{\rm SM}$, is induced by the fourth sequential neutrino ν'. We can use the above equation to get parameter space of $f(x)$ and $V_{2\nu'}$ to $m_{\nu'}$ (see Fig. 3 and 4),

$$(V_{2\nu'})^2 = \frac{\sqrt{2}\cdot\delta a_\mu^{\rm SM}}{8G_{\rm F}\alpha m_\mu^2\cdot f(x)} \tag{13}$$

From Fig. 3, we can see that $f(x)$ is taken negative values except for a very narrow range of $m_{\nu'}$ which is from 58 GeV to 80 GeV. In other words, the sign of $a_\mu^{\rm SM4}$ is only related to ν mass. Only in this narrow mass range, ν' gives positive contribution to $g_\mu-2$. The low bound of $m_{\nu'}$ we get from $g_\mu-2$ is consistent with the present experiments [4]. Let's look at the parameter space of the MNS matrix element factors again. Considering the constraint from $g_\mu-2$, we see that the upper bounds of $m_{\nu'}$ don't exceed 80 GeV. Although the low bound of $m_{\nu'}$ we get from $l\to l'\gamma$ is consistent with the present experiments [4], the upper bound of $m_{\nu'}$, $m_{\nu'}<80$GeV, seems to conflict with the current experiments statue which there is no any new physics signals upper to several GeVs. The fourth generation particles seems not to be so light. They should be several hundred GeVs weight.Moreover, from Fig. 4, if we consider the unitarity of the matrix $V_{\rm MNS}^{SM4}$ and tiny values of its elements, the reasonable value range of m'_ν will be more narrow.

In summary, we calculate the contribution of the fourth generation to $g-2$ and LFV decays and get an interesting result which we can exclude most values of $m_{\nu'}$. Considering the current experimental bounds, we give the parameter space of $m_{\nu'}$ and lepton mixing matrix element $V_{l\nu'}$. We find that LFV processes can constrain on the neutrino mass of the fourth generation: i.e. its mass should be lighter than 80 GeV. It seems that from the lepton part, the current experiments can impose a stringent constraint on the existence of the fourth generation.

Acknowledgments

This research is supported by the the Chinese Postdoctoral Science Foundation and CAS K.C. Wong Postdoctoral Research Award Fund. We are grateful to prof. C.S. Huang and Prof. X.M Zhang for useful discussions.

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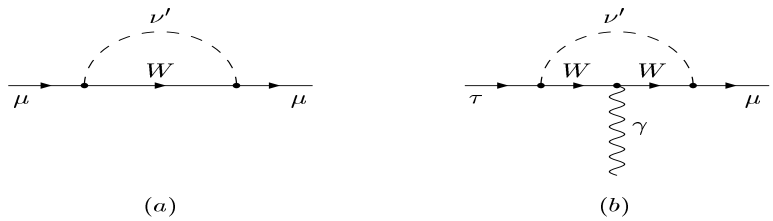


Figure 1: Feynman diagrams for (a) ν anomalous magnetic moment and (b) LFV decays with the fourth generation.

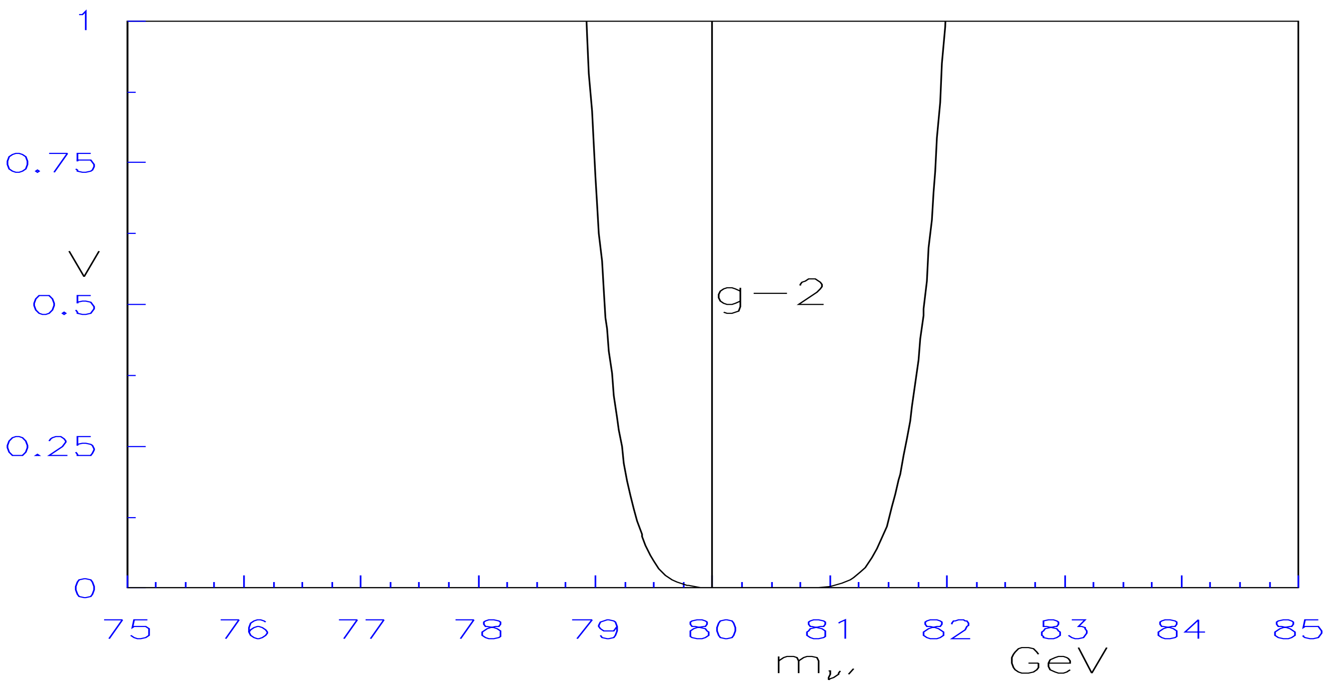


Figure 2: Diagram of $|V_{1\nu'}V_{2\nu'}|^2$, $|V_{1\nu'}V_{3\nu'}|^2$ and $|V_{3\nu'}V_{2\nu'}|^2$ to $m_{\nu'}$

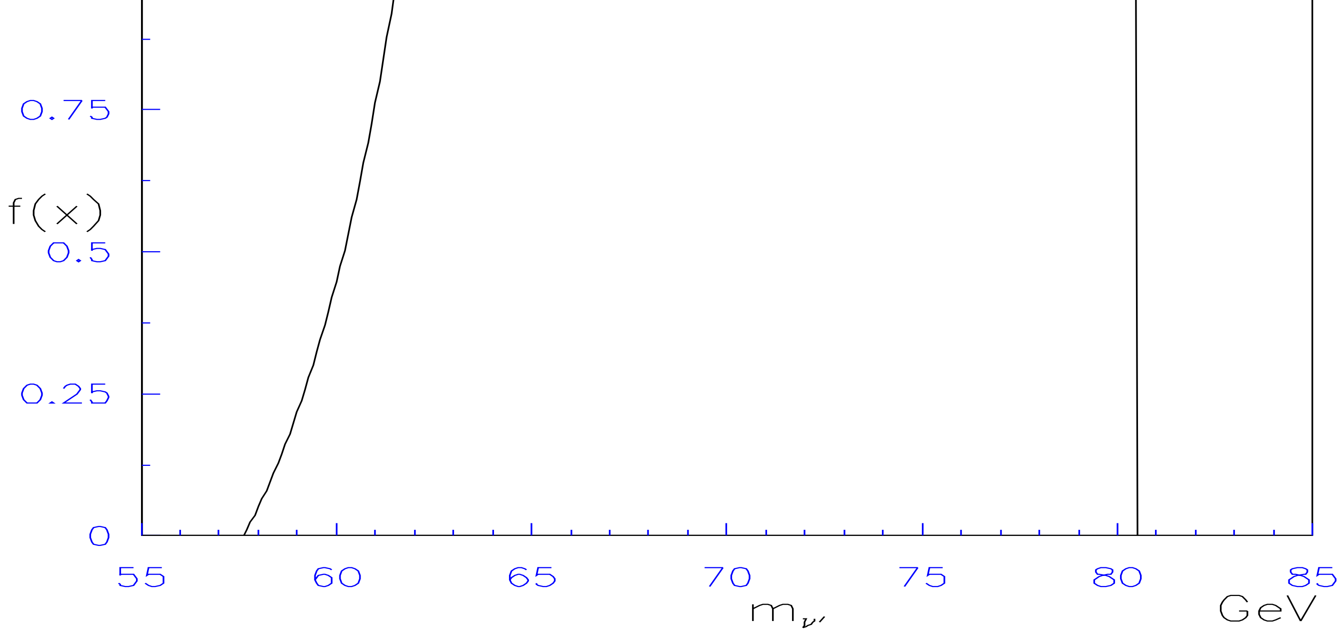


Figure 3: Diagram of f(x) to $m_{\mu'}$

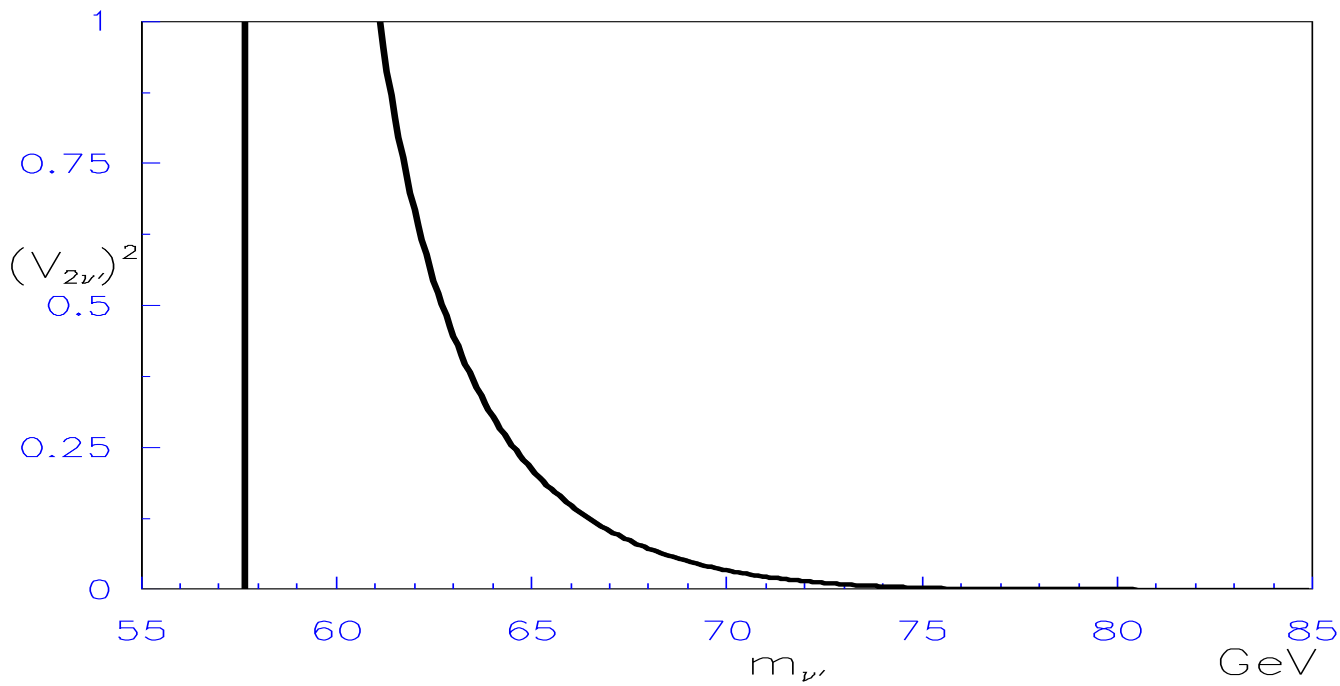
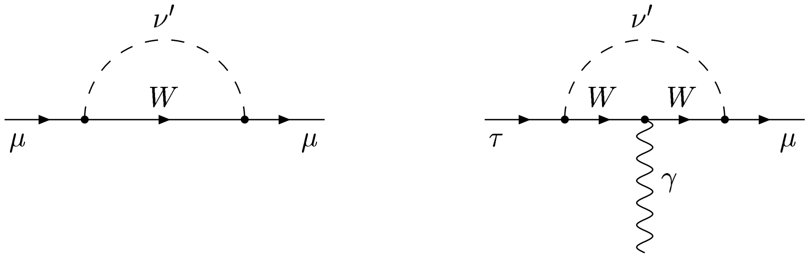


Figure 4: Diagram of$V_{2\mu'}$ to $m_{\mu'}$.



(a) (b)